# Repetition

**Submission deadline: July $28^{\text{th}}$ 2023**

Determine the function $f(n)$ such that the $n^{th}$ term of the sequence

$$1, 2, 2, 3, 3, 3, 4, 4, 4, 4, \cdots$$

is given by $f(n)$.

The problem was solved by

- Merdangeldi Bayramov, *Turkmenistan.*

- Shaher EBRAHEEM, *Egypt.*

- Teawoo Kim, *St. Paul's School, Concord, NH, USA.*

- Ruben Victor Cohen, *Argentina.*

Let $f(n) = k$. Then we have

$$1, 2, 2, \cdots, (k-1), (k-1), \cdots, (k-1), k, \cdots, k, (k+1), (k+1), \cdots$$

and

$$\frac{1}{2}k(k-1) \leq n$$

Solving the equation $k^2 - k + 2 = 2n$, gives us the positive root $\alpha = \frac{1}{2}(1 + \sqrt{8n-7})$. It is not difficult to see that the polynomial $p(x) = x^2 - x + 2 - 2n > 0$ if $x > \alpha$ therefore the desired value of $f(n)$ is the integer part of $\frac{1}{2}(1 + \sqrt{8n-7})$.