

Recurring Differences

Submission deadline: March 30th 2020

Let a_1, a_2, \dots, a_{44} be 44 natural numbers such that

$$0 < a_1 < a_2 < \dots < a_{44} \leq 125.$$

Prove that at least one of the 43 differences $d_j = a_{j+1} - a_j$, occurs at least 10 times.

The problem was solved by

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Discussion:

Assume, to the contrary, that no value repeats more than 9 times. Then,

$$d_1 + d_2 + \cdots + d_{43} \geq 9 \cdot 1 + 9 \cdot 2 + 9 \cdot 3 + 9 \cdot 4 + 5 \cdot 7$$

Thus $d_1 + d_2 + \cdots + d_{43} \geq 125$. But $d_1 + d_2 + \cdots + d_{43} = a_{44} - a_1$. Thus we have that $a_{44} - a_1 \geq 125$, hence $a_{44} > 125$. Therefore, our assumption must be wrong.