Sum of Sines

Submission deadline: May 31^{st} 2018

Let α, β and γ denote the angles of a triangle. Show that

$$\sin(\alpha) + \sin(\beta) + \sin(\gamma) = 4\cos\left(\frac{\alpha}{2}\right)\cos\left(\frac{\beta}{2}\right)\cos\left(\frac{\gamma}{2}\right)$$

The problem was solved by

- Lina Ghonim, *Al Mawakeb School Garhoud, UAE.*
- Kamel Samara, *College of Medicine, Univerity of Sharjah, UAE.*
- Khaled Fahmy Afifi, *American Univeristy of Sharjah, UAE.*
- Issam Louhichi, *American Univeristy of Sharjah, UAE.*
- Erhan Tahir.

Discussion:
Clearly,

$$
\begin{aligned}
\sin(\alpha)+\sin(\beta)+\sin(\gamma) &= \sin(\alpha)+\sin(\beta)+\sin(\pi-(\alpha+\beta)) \\
&= \sin(\alpha)+\sin(\beta)+\sin(\alpha+\beta) \\
&= \sin(\alpha)+\sin(\beta)+\sin(\alpha)\cos(\beta)+\cos(\alpha)\sin(\beta) \\
&= \sin(\alpha)(1+\cos(\beta))+\sin(\beta)(1+\cos(\alpha)) \\
&= 2\left(\sin(\alpha)\cos^2\left(\frac{\beta}{2}\right)+\sin(\beta)\cos^2\left(\frac{\alpha}{2}\right)\right) \\
&= 4\cos\left(\frac{\alpha}{2}\right)\cos\left(\frac{\beta}{2}\right)\left(\sin\left(\frac{\alpha}{2}\right)\cos\left(\frac{\beta}{2}\right)+\sin\left(\frac{\beta}{2}\right)\cos\left(\frac{\alpha}{2}\right)\right) \\
&= 4\cos\left(\frac{\alpha}{2}\right)\cos\left(\frac{\beta}{2}\right)\sin\left(\frac{\alpha+\beta}{2}\right) \\
&= 4\cos\left(\frac{\alpha}{2}\right)\cos\left(\frac{\beta}{2}\right)\cos\left(\frac{\gamma}{2}\right)
\end{aligned}
$$