Sum of Sines

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Let n be a natural number. Find

$$\sin(1) + \sin(2) + \sin(3) + \cdots + \sin(n)$$

The problem was solved by

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Discussion;

From Eulers formula it follows that

$$e^{im} = \cos(m) + i\sin(m)$$

where $i^2 = -1$, and m is a real number. Thus

$$e^i + e^{i2} + \cdots + e^{in} = \cos(1) + \cdots + \cos(n) + i[\sin(1) + \cdots + \sin(n)]$$

Therefore, the imaginary part of $e^i + e^{i2} + \cdots + e^{in}$ is equal to $\sin(1) + \cdots + \sin(n)$.

The sum $e^i + e^{i2} + \cdots + e^{in}$ is a geometric series whose ratio is e^i therefore,

$$e^i + e^{i2} + \cdots + e^{in} = e^i \frac{1 - e^{in}}{1 - e^i}$$

Now by computing the imaginary part of $e^i \frac{1 - e^{in}}{1 - e^i}$ we get

$$\sin(1) + \sin(2) + \cdots + \sin(n) = \frac{\sin(1) - \sin(n+1) + \sin(n)}{2(1 - \cos(1))}$$